Assignment 2

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Contents

1 Exercise 1 : Bayes Ball

Nodes F and G are reachable from A, given the evidence. As we don't know F, we cannot go to B.

2 Exercise 2 : Variable Elimination

Elimination order list : A, B, E, C, D.
Original active factor list: p (A), p (B), p (E / C), p (C / A , B), p (D / C)

2.1 Eliminate A

$$m_A(B,C) = \sum_a p(a)p(c/a,b) = p(C/B)$$

Factor list: p (B), p (E / C), p (D / C), $m_A(B,C)$

2.2 Eliminate B

$$m_B(C) = \sum_b p(b) m_A(b, c) = p(C)$$

Factor list: p (E / C), p (D / C), $m_B(C)$

2.3 Eliminate E

$$m_E(C) = \sum_e p(e/c) = 1$$

Factor list: p (D / C), $m_B(C)$, $m_E(C)$

2.4 Eliminate C

$$m_C(D) = \sum_c p(d/c) m_B(c) m_E(c) = p(D)$$

Factor list: $m_C(D)$

2.5 Eliminate D

$$m_D = \sum_d m_c(d) = 1$$

Factor list: m_D

As you can see, the variable E is relevant in the computation of p (D): we wouldn't get the same result if we didn't take into account the variable E.

3 Exercise 3 : Directed vs Undirected models

3.1 Model A

The model implies that B and C are independent given A.

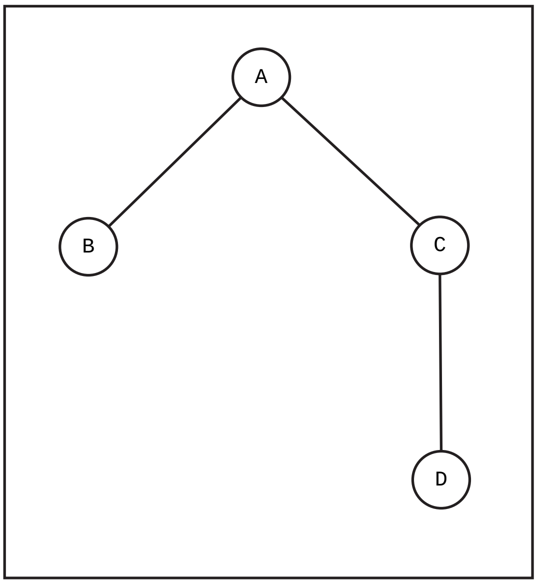

Figure 1: Undirected model

3.2 Model B

The model implies that B and C are independent given A.

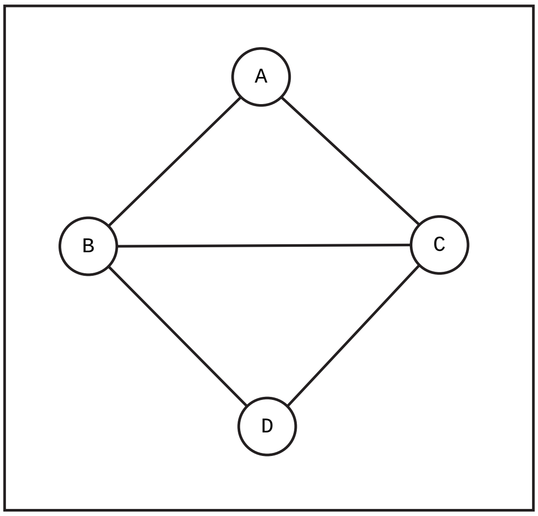

Figure 2: Undirected model

4 Exercise 4 : Undirected models

To capture any joint distribution between any couples of 2 nodes, we need a fully connected graph: with an arc between any 2 nodes. The maximal clique contains all the nodes A, B, C and D.

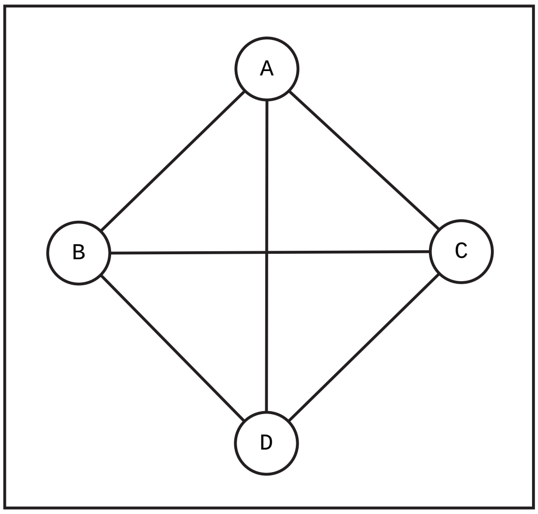

Figure 3: Undirected model

To know how many edges we need to connect n nodes, let us imagine that we have a matrix n by n, where the intersection tells us that there can be an edge between variable i and variable j. We wouldn't need half of the matrix as the model is undirected. In the $\frac{n^2}{2}$ other possibilities, we have to remove the diagonal, which contains arcs from i to i. We finally need $\frac{n^2}{2} - n$ edges to capture any joint distribution, if we have n variables.

5 Exercise 5 : Junction Tree

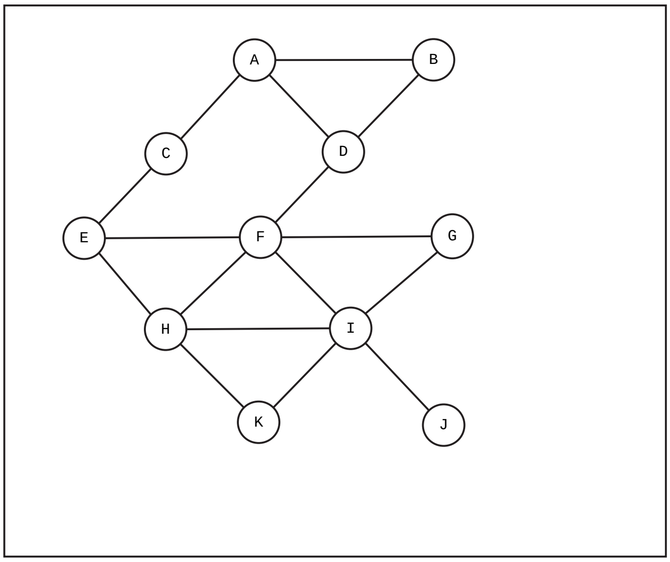

Figure 4: Moralized Graph

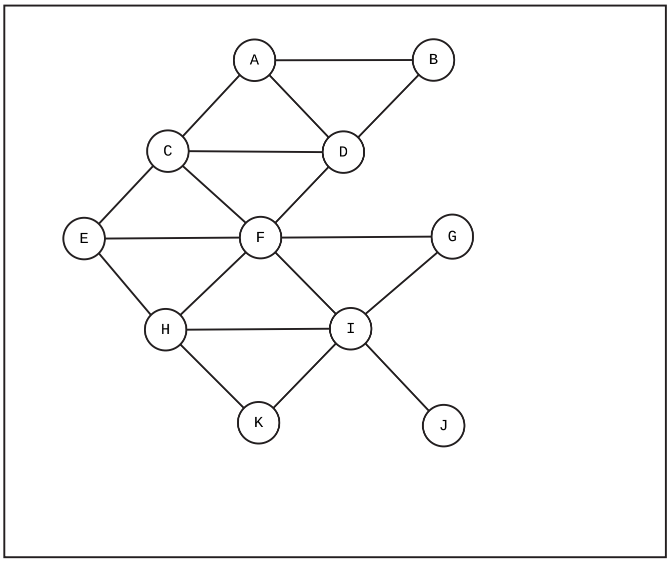

Figure 5: Triangulated Graph

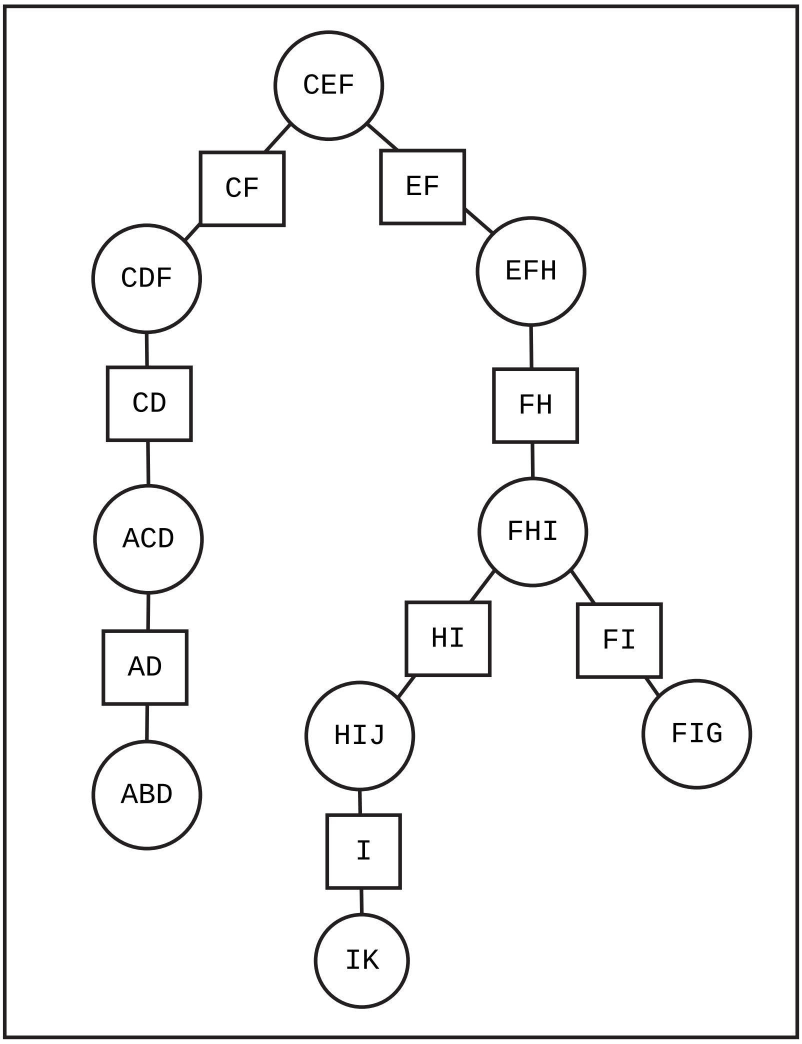

Figure 6: Junction Tree

6 Exercise 6 : Clique trees

In the new network, at least we keep the clique size at k. Let us now consider that the maximum clique size in the new network is k+2. We would have for k = 3 :

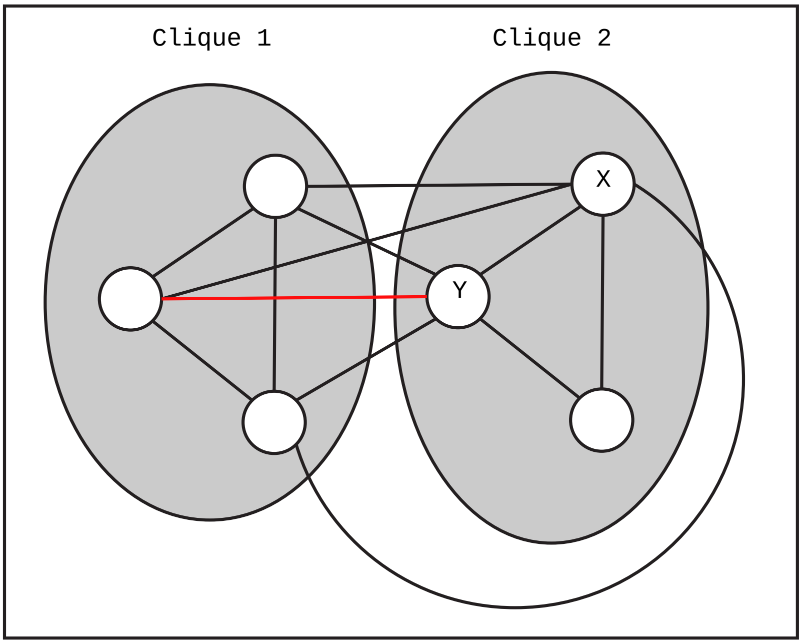

Figure 7: New network with k=3+2

When we add the red arc to add nodes X and Y to the clique 1, we obtain a maximum clique of k=k+2, but the previous maximum clique size was not 3. Indeed Y can be added to the clique 1 in order to form a bigger clique, which the size would be 4.

With this simple eample, we can show that by adding one arc to a network, the maximum clique size can only be incremented by one.

7 Exercise 7 : I-Maps

By reversing the arc from BA to AB, we losintroduce the conditional independency between E and B given A. With adding an arc from E to B, we ensure that the new network is an I-map of the original one.

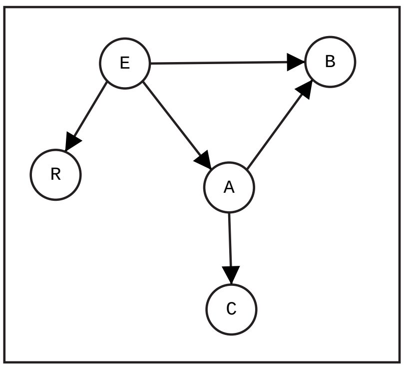

Figure 8: I-map