Assignment 1

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1 Exercise 1

$$X \amalg Y/Z \Rightarrow p(X/Y,Z) = p(X/Z)$$

$$p(X,Y,Z) = p(Z) \times p(X/Z) \times p(Y/Z)$$

2 Exercise 2 : Conditional probability warm-up

$$\begin{aligned} p(A,B/E) &= \frac{p(A,B,E)}{p(E)} \\ &= \frac{p(A/B,E) \times p(B,E)}{p(E)} \\ &= \frac{p(A/B,E) \times p(B/E) \times p(E)}{p(E)} \\ &= p(A/B,E) \times p(B/E) \end{aligned}$$

$$\begin{aligned} p(A/B,C) &= \frac{p(A,B,C)}{p(B,C)} \\ &= \frac{p(C)\times p(A,B/C)}{p(C)\times p(B/C)} \\ &= \frac{p(B/A,C)\times p(A/C)}{p(B/C)} \end{aligned}$$

3 Exercise 3 : Conditional probability

$$\begin{aligned} p(A,B/C) &= p(A/B,C)\times p(B/C) \\ &= p(A/C)\times p(B/C) \end{aligned}$$

$$\begin{aligned} A \amalg B/C \Rightarrow p(A,B/C) &= p(A/C)\times p(B/C) \\ &= p(B/C)\times p(A/C) \\ &= p(B,A/C) \Rightarrow B \amalg A/C \end{aligned}$$

$$\begin{aligned} A \amalg (B\cup D)/C &\Rightarrow p(A/(B\cup D),C) = p(A/C) \\ &\Rightarrow \begin{cases} p(A/B,C)=p(A,C) \Rightarrow A \amalg B/C \\ \text{and} \\ p(A/D,C)=p(A,C) \Rightarrow A \amalg D/C \end{cases} \end{aligned}$$

$$\begin{aligned} A \amalg (B\cup D)/C &\Rightarrow \begin{cases} A \amalg B/C \Rightarrow p(A/B,C)=p(A,C) \\ A \amalg D/C \Rightarrow p(A/D,C)=p(A,C) \end{cases} \\ &\Rightarrow \begin{cases} p(A/B,C)=p(A,C) \Rightarrow p(A/B,C\cup D)=p(A,C\cup D) \\ p(A/D,C)=p(A,C) \Rightarrow p(A/D,B\cup C)=p(A,B\cup C) \end{cases} \\ &\Rightarrow \begin{cases} A \amalg B/C\cup D \\ A \amalg D/B\cup C \end{cases} \end{aligned}$$

4 Exercise 4

4.1 With no information

$$p(H/E_1,E_2) = \frac{p(E_1,E_2/H)\times p(H)}{p(E_1,E_2)}$$

We need the ii sets of numbers.

4.2 With conditional independence

$$p(H/E_1,E_2) = \frac{p(E_1/H)\times p(E_2/H)\times p(H)}{p(E_1,E_2)}$$

We need the i sets of numbers.

4.3 With boolean variables

We need 3 numbers for the first case and only 2 ($p(H)$ and $p(E_1, E_2)$) for the second one.

5 Exercise 5

5.1 Bayes network

Boolean variables:

- A: "Alien wears a weapon"
- B: "The weapon is a neurotransmitter disruptor"
- C: "Data is disabled or damaged"
- D: "The alien fires his weapon"
- E: "Data out-powers the alien"
- F: "Data escapes"

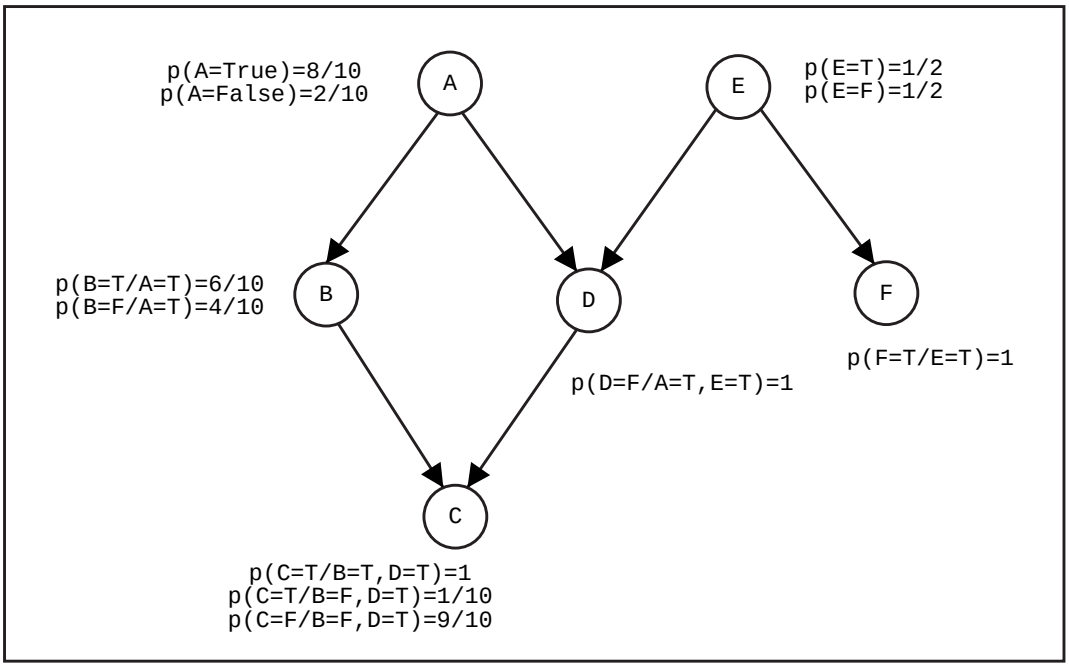

Figure 1: Bayes network of Data

5.2 Without information

$$\begin{aligned} p(C) &= p(C/B)p(B) + p(C/D)p(D) \\ &= p(C/B)p(B/A)p(A) + p(C/D)\left[p(D/A)p(A) + p(D/E)p(E)\right] \end{aligned}$$

5.3 Given that the alien has a weapon

$$p(C/A=T) = p(C/B)p(B/A=T) + p(C/D)\left[p(D/A=T) + p(D/E)p(E)\right]$$

5.4 Given that Data escapes

$$p(A/F=T) = \frac{p(F/A)p(A)}{p(F)}$$