Assignment 4

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1 Exercise 1

1.1 Phase 1

$Node$	X_{75}	X_{61}	X_{63}	X_{62}	X_{67}	X_{13}	X_{21}	X_{23}	X_{14}	X_{15}	X_{54}	X_{24}	X_{25}	X_{72}
1	0	1	0	0	0	-1	1	0	-1	-1	0	0	0	0
2	0	0	0	1	0	0	-1	-1	0	0	0	-1	-1	1
3	0	0	1	0	0	1	0	1	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	1	0	1	1	0	0
5	1	0	0	0	0	0	0	0	0	1	-1	0	1	0
6	0	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0
7	-1	0	0	0	1	0	0	0	0	0	0	0	0	-1

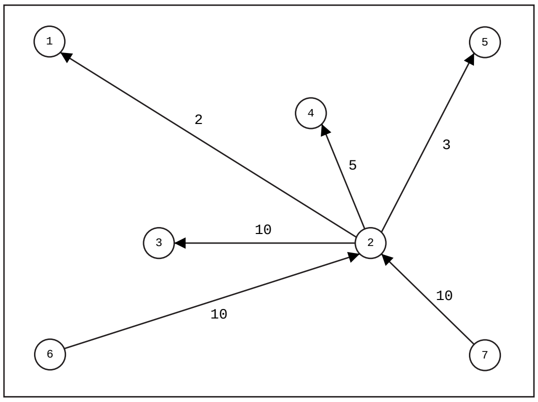
We can eliminate the first line to obtain the initial dictionary of the first phase :

$$\left\{ \begin{array}{rcl} X_{62} - X_{21} - X_{23} - X_{24} - X_{25} + X_{72} & = & 0 \\ X_{63} + X_{13} + X_{23} + X_0 & = & 10 \\ X_{14} + X_{54} + X_{24} + X_0 & = & 5 \\ X_{75} + X_{15} - X_{54} + X_{25} + x_0 & = & 3 \\ -X_{61} - X_{63} - X_{62} - X_{67} - X_0 & = & -10 \\ -X_{75} + X_{67} - X_{72} - X_0 & = & -10 \end{array} \right.$$

After minimizing X_0, we obtain this feasible solution :

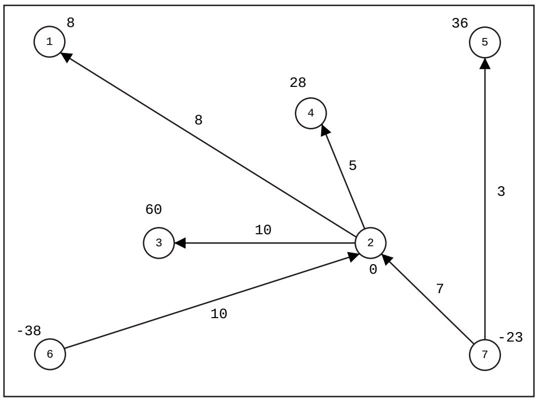
$$\begin{cases} X_{75} &= 0 \\ X_{61} &= 0 \\ X_{63} &= 0 \\ X_{62} &= 10 \\ X_{67} &= 0 \\ X_{13} &= 0 \\ X_{21} &= 2 \\ X_{23} &= 10 \\ X_{14} &= 0 \\ X_{15} &= 0 \\ X_{54} &= 0 \\ X_{24} &= 5 \\ X_{25} &= 3 \\ X_{72} &= 10 \end{cases}$$

representing this tree :

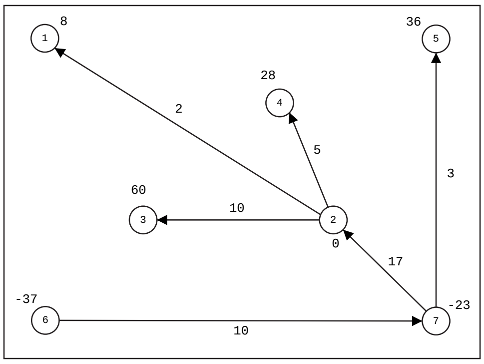


1.2 Phase 2

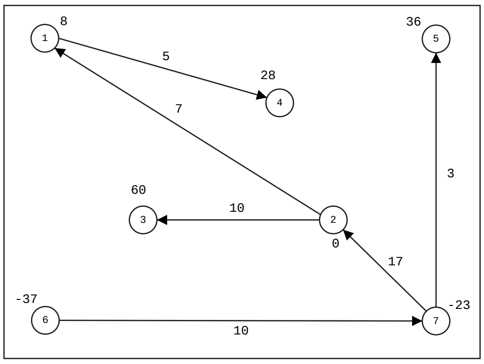
After entering X_{75} and leaving X_{25}, the tree becomes :



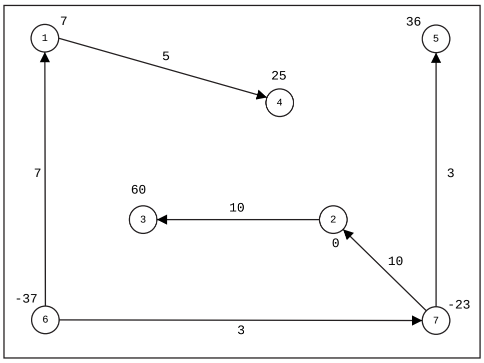
After entering X_{67} and leaving X_{62}, the tree becomes :



After entering X_{14} and leaving X_{24}, the tree becomes :



After entering X_{61} and leaving X_{21}, the tree becomes :



This tree is optimum, there does not exist any improvement.

2 Exercise 2

3 Exercise 3

$A(X)/B(Y)$	$(1,1)$	$(1,2)$	$(1,3)$	$(2,1)$	$(2,2)$	$(2,3)$	$(3,1)$	$(3,2)$	$(3,3)$
$(1,1)$	0	1	1	-1	0	0	-1	0	0
$(1,2)$	-1	0	0	1	2	2	-1	0	0
$(1,3)$	-1	0	0	-1	0	0	2	3	3
$(2,1)$	1	-1	1	0	-2	0	0	-2	0
$(2,2)$	0	-2	0	2	0	2	0	-2	0
$(2,3)$	0	-2	0	0	-2	0	3	1	3
$(3,1)$	1	1	-2	0	0	-3	0	0	-3
$(3,2)$	0	0	-3	2	2	-1	0	0	-3
$(3,3)$	0	0	-3	0	0	-3	3	3	0
$\sum_{i=1}^{9} X_i$	0	-3	-6	3	0	-3	6	3	0

So we have to maximize $Z = X_1 + X_4 - 2X_7 - 3X_8 - 3X_9$, considering the following constraints :

$$\left\{\begin{array}{rcl} Z + X_2 + X_3 - X_4 - X_7 & \leq & 0 \\ Z - X_1 + X_4 + 2X_5 + 2X_6 - X_7 & \leq & 0 \\ Z + X_1 - X_2 + X_3 - 2X_5 - 2X_8 & \leq & 0 \\ Z - 2X_2 + 2X_4 + 2X_6 - 2X_8 & \leq & 0 \\ Z - 2X_2 - 2X_5 + 3X_7 + X_8 + 3X_9 & \leq & 0 \\ Z + X_1 + X_2 - 2X_3 - 3X_6 - 3X_9 & \leq & 0 \\ Z - 3X_3 + 2X_4 + 2X_5 - X_6 - 3X_9 & \leq & 0 \\ Z - 3X_3 - 3X_6 + 3X_7 + 3X_8 & \leq & 0 \\ \sum_{i=1}^{9} X_i & = & 1 \end{array}\right.$$

which leads us to the primal optimal solution :

$$\left\{\begin{array}{rcl} X_1 & = & \frac{3}{11} \\ X_2 & = & \frac{3}{11} \\ X_3 & = & 0 \\ X_4 & = & \frac{3}{11} \\ X_5 & = & 0 \\ X_6 & = & 0 \\ X_7 & = & 0 \\ X_8 & = & 0 \\ X_9 & = & \frac{2}{11} \\ Z & = & 0 \end{array}\right.$$

The dual optimal solution is :

$$\left\{\begin{array}{rcl} Y_1 & = & \frac{6}{11} \\ Y_2 & = & 0 \\ Y_3 & = & \frac{1}{11} \\ Y_4 & = & 0 \\ Y_5 & = & \frac{2}{11} \\ Y_6 & = & 0 \\ Y_7 & = & \frac{2}{11} \\ Y_8 & = & 0 \\ Y_9 & = & 0 \\ W & = & 0 \end{array}\right.$$