Assignment 2

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Contents

1 Exercise 1

1.1 Initial dictionary - Phase 1

$$\left\{\begin{array}{rcl} X_6 & = & 6 - X_1 - X_2 - X_3 + X_4 - 2X_5 + X_0 \\ X_7 & = & -9 - X_1 + 2X_2 + X_3 + X_4 + X_5 + X_0 \\ Z & = & -X_0 \end{array}\right.$$

1.2 Pivots

Leaving	Entering
X_7	X_0
X_6	X_3
X_0	X_2

1.3 Final dictionary - Phase 1

$$\begin{cases} X_2 &=& 3+2X_1-2X_4+X_5+X_6+X_7-2X_0 \\ X_3 &=& 3-3X_1+3X_4-3X_5-2X_6-X_7+3X_0 \\ Z &=& 0-X_0 \end{cases}$$

1.4 Initial dictionary - Phase 2

$$\begin{cases} X_2 &=& 3+2X_1-2X_4+X_5+X_6+X_7 \\ X_3 &=& 3-3X_1+3X_4-3X_5-2X_6-X_7 \\ Z &=& -15-4X_1+3X_4+2X_5-2X_6-3X_7 \end{cases}$$

1.5 Pivots

Leaving	Entering
X_3	X_5
X_2	X_4

1.6 Final dictionary - Phase 2

$$\begin{cases} X_4 &=& 4+X_1-X_2-\frac{1}{3}X_3+\frac{1}{3}X_6+\frac{2}{3}X_7 \\ X_5 &=& 5-X_2-\frac{2}{3}X_3-\frac{1}{3}X_6+\frac{1}{3}X_7 \\ Z &=& 7-X_1-5X_2-\frac{7}{3}X_3-\frac{5}{3}X_6-\frac{1}{3}X_7 \end{cases}$$

1.7 Optimum solution for primal

$$\begin{cases} X_1 &=& X_2=X_3=0 \\ X_4 &=& 4 \\ X_5 &=& 5 \\ Z^* &=& 7 \end{cases}$$

1.8 Dictionary and optimum solution for dual

$$\begin{cases} Y_1+Y_2 &\geq& 1 \\ Y_1-2Y_2 &\geq& -4 \\ Y_1-Y_2 &\geq& -1 \\ -Y_1-Y_2 &\geq& -2 \\ 2Y_1-Y_2 &\geq& -3 \\ W^* &=& 6Y_1-9Y_2 \end{cases} \Rightarrow \begin{cases} Y_1 &=& \frac{5}{3} \\ Y_2 &=& \frac{1}{3} \\ W^* &=& 7 \end{cases}$$

1.9 Duality Theorem

The duality thorem is verified since $Z^* = W^* = 7$.

2 Exercise 2

2.1 Dictionary and optimum solution for primal

Let X_1 be the number of fresh hams, X_2 the number of hams smoked on regular time, X_3 the number of hams smoked on overtime, X_4 the number of fresh pork bellies, ...so the initial dictionary is :

$$\begin{cases} X_1 + X_2 + X_3 = 480 \\ X_4 + X_5 + X_6 = 400 \\ X_4 + X_5 + X_6 = 400 \\ X_2 + X_5 + X_8 \leq 420 \\ X_3 + X_6 + X_9 \leq 250 \\ Z = 8X_1 + 14X_2 + 11X_3 + 4X_4 + 12X_5 + 7X_6 + 4X_7 + 13X_8 + 9X_9 \end{cases}$$

which gives the following solution :

$$\begin{cases} X_1^* & = & 440 \\ X_2^* & = & 0 \\ X_3^* & = & 40 \\ X_4^* & = & 0 \\ X_5^* & = & 400 \\ X_6^* & = & 0 \\ X_7^* & = & 0 \\ X_8^* & = & 20 \\ X_9^* & = & 210 \\ Z^* & = & 10910 \end{cases}$$

2.2 Dictionary and optimum solution for dual

$$\begin{cases} Y_1 \geq 8 \\ Y_1 + Y_4 \geq 14 \\ Y_1 + Y_5 \geq 11 \\ Y_2 \geq 4 \\ Y_2 + Y_4 \geq 12 \\ Y_2 + Y_5 \geq 7 \\ Y_3 \geq 4 \\ Y_3 + Y_4 \geq 13 \\ Y_3 + Y_5 \geq 9 \\ W = 480Y_1 + 400Y_2 + 230Y_3 + 420Y_4 + 250Y_5 \end{cases} \Rightarrow \begin{cases} Y_1^* & = & 8 \\ Y_2^* & = & 5 \\ Y_3^* & = & 6 \\ Y_4^* & = & 7 \\ Y_5^* & = & 3 \\ W^* & = & 10910 \end{cases}$$

2.3 Theorems verification

2.3.1 Duality

The duality theorem is verified since $Z^* = W^* = 10910$.

2.3.2 Complementary slackness conditions

System 1 :

$$\begin{cases} j & = & 1 \rightarrow \sum_{i=1}^m a_{i,1}Y_i^* = 1 \times 8 + 0 \times 5 + 0 \times 6 + 0 \times 7 + 0 \times 3 = 8 = c_1 \\ j & = & 2 \rightarrow \sum_{i=1}^m a_{i,2}Y_i^* = 1 \times 8 + 0 \times 5 + 0 \times 6 + 1 \times 7 + 0 \times 3 = 15 \neq c_1 \text{ but } X_2^* = 0 \\ j & = & 3 \rightarrow \sum_{i=1}^m a_{i,3}Y_i^* = 1 \times 8 + 0 \times 5 + 0 \times 6 + 0 \times 7 + 1 \times 3 = 11 = c_3 \\ j & = & 4 \rightarrow \sum_{i=1}^m a_{i,4}Y_i^* = 0 \times 8 + 1 \times 5 + 0 \times 6 + 0 \times 7 + 0 \times 3 = 5 \neq c_4 \text{ but } X_4^* = 0 \\ j & = & 5 \rightarrow \sum_{i=1}^m a_{i,5}Y_i^* = 0 \times 8 + 1 \times 5 + 0 \times 6 + 1 \times 7 + 0 \times 3 = 12 = c_5 \\ j & = & 6 \rightarrow \sum_{i=1}^m a_{i,6}Y_i^* = 0 \times 8 + 1 \times 5 + 0 \times 6 + 0 \times 7 + 1 \times 3 = 8 \neq c_6 \text{ but } X_6^* = 0 \\ j & = & 7 \rightarrow \sum_{i=1}^m a_{i,7}Y_i^* = 0 \times 8 + 0 \times 5 + 1 \times 6 + 0 \times 7 + 0 \times 3 = 6 \neq c_7 \text{ but } X_7^* = 0 \\ j & = & 8 \rightarrow \sum_{i=1}^m a_{i,8}Y_i^* = 0 \times 8 + 0 \times 5 + 1 \times 6 + 1 \times 7 + 0 \times 3 = 13 = c_8 \\ j & = & 9 \rightarrow \sum_{i=1}^m a_{i,9}Y_i^* = 0 \times 8 + 0 \times 5 + 1 \times 6 + 0 \times 7 + 1 \times 3 = 9 = c_9 \end{cases}$$

and System 2 :

$$\begin{cases} i &=& 1 \rightarrow \sum_{j=1}^{n} a_{1,j} X_j^* = 1 \times 440 + 1 \times 40 + 0 \times 400 + 0 \times 20 + 0 \times 210 = 480 = b_1 \\ i &=& 2 \rightarrow \sum_{j=1}^{n} a_{2,j} X_j^* = 0 \times 440 + 0 \times 40 + 1 \times 400 + 0 \times 20 + 0 \times 210 = 400 = b_2 \\ i &=& 3 \rightarrow \sum_{j=1}^{n} a_{3,j} X_j^* = 0 \times 440 + 0 \times 40 + 0 \times 400 + 1 \times 20 + 1 \times 210 = 230 = b_3 \\ i &=& 4 \rightarrow \sum_{j=1}^{n} a_{4,j} X_j^* = 0 \times 440 + 0 \times 40 + 1 \times 400 + 1 \times 20 + 0 \times 210 = 420 = b_4 \\ i &=& 5 \rightarrow \sum_{j=1}^{n} a_{5,j} X_j^* = 0 \times 440 + 1 \times 40 + 1 \times 400 + 0 \times 20 + 1 \times 210 = 250 = b_5 \end{cases}$$

So the theorem 5.2 is verified.

Concerning the theorem 5.3, thanks to System 1, we have proved that :

$$\sum_{i=1}^{m} a_{i,1} Y_i^* = c_j \text{ whenever } X_j^* > 0$$

and

$$\sum_{i=1}^{m} a_{i,1} Y_i^* \geq c_j \text{ for all } j = 1, 2 \cdots, n$$

2.4 Economic interpretation

First let me remember the different dimensions of our data : X is a number of product, b also, c is a profit by product, a is a coefficient and has no dimension. If we use the formula :

$$\sum_{i=1}^{m} a_{i,1} Y_i^* = c_j$$

Y is analog to c : a profit by product.

3 Exercise 3