Assignment 1

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1 Exercise 1

Assume that X_i is the number of assemblers during the i-th week ($i = 1, 2, 3, 4$). During the first week, we have only 40 workers, so :

$$X_1 \leq 40$$

and we have $3(40 - X_1)$ trainees. During the 2nd week, we have a workforce of $40 + 3(40 - X_1) = 160 - 3X_1$ employees. So :

$$X_2 \leq 160 - 3X_1$$

and we have $3(160 - 3X_1 - X_2)$ trainees. During the 3rd week, we have $(160 - 3X_1) + 3[(160 - 3X_1) - X_2] = 640 - 12X_1 - 3X_2$ employees. So :

$$X_3 \leq 640 - 12X_1 - 3X_2$$

and we have $3(640 - 12X_1 - 3X_2 - X_3)$ trainees. During the 4th week, we have $(640 - 12X_1 - 3X_2) + 3[(640 - 12X_1 - 3X_2) - X_3] = 2560 - 48X_1 - 12X_2 - 3X_3$ employees. So :

$$X_4 \leq 2560 - 48X_1 - 12X_2 - 3X_3$$

and we do not have trainees because it is the last week. Workers, who do not work, are idle. The contract say that the company has to produce 20 000 radios. So :

$$50(X_1 + X_2 + X_3 + X_4) = 20000 \Rightarrow X_1 + X_2 + X_3 + X_4 = 400$$

The cost of radios for the 4 weeks is : $50 \times 20000 = 100000$. The radios will be sold as follows :

$$\begin{aligned} Sales &= 20 \times 50 \times X_1 + 18 \times 50 \times X_2 + 16 \times 50 \times X_3 + 14 \times 50 \times X_4 \\ &= 1000X_1 + 900X_2 + 800X_3 + 700X_4 \end{aligned}$$

Now, we have to calculate the wages in order to calculate the profit. During each week, the wages will be :

$$\begin{array}{rcl} 1^{st}\text{week} & : & 200 \times 40 + 100 \times 3(40 - X_1) \\ 2^{nd}\text{week} & : & 200(160 - 3X_1) + 100 \times 3(160 - 3X_1 - X_2) \\ 3^{rd}\text{week} & : & 200(640 - 12X_1 - 3X_2) + 100 \times 3(640 - 12X_1 - 3X_2 - X_3) \\ 3^{rd}\text{week} & : & 200(2560 - 48X_1 - 12X_2 - 3X_3) \end{array}$$

When we add everything, we obtain the total amount of wages, which is :

$$Wages = 932000 - 17400x_1 - 4200X_2 - 900X_3$$

The formula for the profit is : Profit = Sales - (Cost + Wages) :

$$Profit = 1000X_1 + 900X_2 + 800X_3 + 700X_4 - (100000 + 932000 - 17400x_1 - 4200X_2 - 900X_3)$$

So we have to maximise the profit function :

$$Profit = -832000 + 18400X_1 + 5100X_2 + 1700X_3 + 700X_4$$

respecting our constraints :

$$\left\{ \begin{array}{lcr} X_1 & \leq & 40 \\ 3X_1 + X_2 & \leq & 160 \\ 12X_1 + 3X_2 + X_3 & \leq & 640 \\ 48X_1 + 12X_2 + 3X_3 + X_4 & \leq & 2560 \\ X_1 + X_2 + X_3 + X_4 & = & 400 \end{array} \right.$$

Thanks to Maple, we obtain the following optimum result :

$$\left\{ \begin{array}{l} X_1 = 10 \\ X_2 = X_3 = X_4 = 130 \end{array} \right.$$

and the maximum profit is 327 000 $.

2 Exercise 2

$$\left\{ \begin{array}{ccl} x_4 & = & -2x_1 + x_2 - x_3 \\ x_5 & = & -3x_1 - x_2 - x_3 \\ x_6 & = & 5x_1 - 3x_2 + 2x_3 \\ Z & = & x_1 - 2x_2 + x_3 \end{array} \right.$$

We use x_1 as the pivot and x_5 get out of the basis. We obtain :

$$\left\{ \begin{array}{ccl} x_1 & = & -\frac{1}{3}x_2 - \frac{1}{3}x_3 - \frac{1}{3}x_5 \\ x_4 & = & \frac{5}{3}x_2 - \frac{1}{3}x_3 + \frac{2}{3}x_5 \\ x_6 & = & -\frac{14}{3}x_2 + \frac{1}{3}x_3 - \frac{5}{3}x_5 \\ Z & = & -\frac{7}{3}x_2 + \frac{2}{3}x_3 - \frac{1}{3}x_5 \end{array} \right.$$

We use x_3 as the pivot and x_4 get out of the basis. We obtain essentially the starting dictionnary back, only the variables have different indices :

$$\left\{ \begin{array}{ccl} x_1 & = & -2x_2 + x_4 - x_5 \\ x_3 & = & 5x_2 - 3x_4 + 2x_5 \\ x_6 & = & -3x_2 - x_4 - 1x_3 \\ Z & = & x_2 - 2x_4 + x_5 \end{array} \right.$$

We use x_2 as the pivot and x_6 get out of the basis. We obtain :

$$\begin{cases} x_1 &= \frac{5}{3}x_4 - \frac{1}{3}x_5 + \frac{2}{3}x_6 \\ x_2 &= -\frac{1}{3}x_4 - \frac{1}{3}x_5 - \frac{1}{3}x_6 \\ x_3 &= -\frac{14}{3}5x_4 + \frac{1}{3}x_5 - \frac{5}{3}x_6 \\ Z &= -\frac{7}{3}x_4 + \frac{2}{3}x_5 - \frac{1}{3}x_6 \end{cases}$$

We use x_5 as the pivot and x_1 get out of the basis. We obtain :

$$\begin{cases} x_2 &= x_1 - 2x_4 - x_6 \\ x_3 &= -x_1 - 3x_4 - x_6 \\ x_5 &= -3x_1 + 5x_4 + 2x_6 \\ Z &= -2x_1 + x_4 + x_6 \end{cases}$$

We use x_4 as the pivot and x_3 get out of the basis. We obtain :

$$\begin{cases} x_2 &= \frac{5}{3}x_1 + \frac{2}{3}x_3 - \frac{1}{3}x_6 \\ x_4 &= -\frac{1}{3}x_1 - \frac{1}{3}x_3 - \frac{1}{3}x_6 \\ x_5 &= -\frac{14}{3}5x_1 - \frac{5}{3}x_3 + \frac{1}{3}x_6 \\ Z &= -\frac{7}{3}x_1 - \frac{1}{3}x_3 + \frac{2}{3}x_6 \end{cases}$$

We use x_6 as the pivot and x_2 get out of the basis. After 6 dictionnaries, we obtain the starting dictionnary :

$$\begin{cases} x_4 &= -2x_1 + x_2 - x_3 \\ x_5 &= -3x_1 - x_2 - x_3 \\ x_6 &= 5x_1 - 3x_2 + 2x_3 \\ Z &= x_1 - 2x_2 + x_3 \end{cases}$$

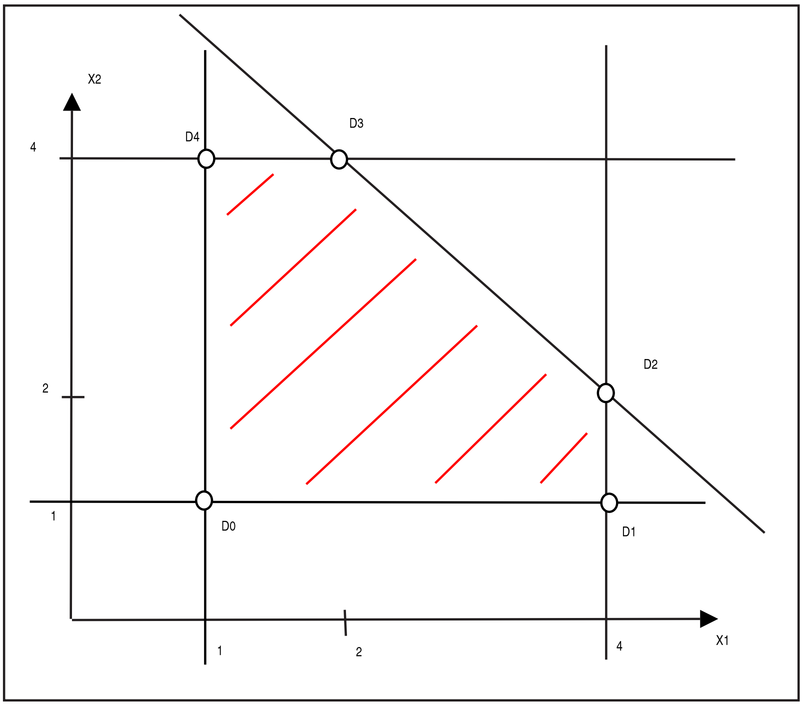
3 Exercise 3

We will prove here that having an optimal dictionnary where the corresponding basic feasible solution has the property that basic variable is strictly positive does not imply that there is a unique optimum solution for the problem.

In order to do that, we will use a counter-example. We have 2 variables X_1 and X_2 and we have to maximize their sum. The constraints are :

$$\begin{cases} 1 \leq X_1 \leq 4 \\ 1 \leq X_2 \leq 4 \\ X_1 + X_2 \leq 6 \\ Z = X_1 + X_2 \end{cases}$$

Geometrically, we have :



So we have an optimal dictionnary (D_0, ..., D_4), where the corresponding solution are basic and feasible and the variables are strictly positive, but we have 2 optimum solutions for the problem : D_2 and D_3 (what we had to prove).