Assignment 4

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Contents

1 Expected value

The formulae is:

$$E(X) = \sum_{i=1}^{n} p_i X_i$$

So for 10 questions we have (0 , 1) with a probability ($\frac{4}{5}$, $\frac{1}{5}$) and the expected value is:

$$E = 10 \times (1 \times \frac{1}{5} + 0 \times \frac{4}{5}) = 2$$

With (- 0,5 , 1), we obtain:

$$E = 10 \times (1 \times \frac{1}{5} - 0,5 \times \frac{4}{5}) = -2$$

The best of action is to not answer: the gain would be zero and it is better than - 2.

With k = 1, we have (- 0,5 , 1) with a probability ($\frac{3}{4}$, $\frac{1}{4}$) and the expected value is:

$$E = 10 \times (1 \times \frac{1}{4} - 0,5 \times \frac{3}{4}) = -\frac{5}{4}$$

With k = 2, we have (- 0,5 , 1) with a probability ($\frac{2}{3}$, $\frac{1}{3}$) and the expected value is:

$$E = 10 \times (1 \times \frac{1}{3} - 0,5 \times \frac{2}{3}) = 0$$

With k = 3, we have (- 0,5 , 1) with a probability ($\frac{1}{2}$, $\frac{1}{2}$) and the expected value is:

$$E = 10 \times (1 \times \frac{1}{2} - 0,5 \times \frac{1}{2}) = \frac{5}{2}$$

2 Utility

2.1 Expected value

The formulae is:

$$E(X) = \sum_{i=1}^{n} p_i X_i$$

The probability to win 2^n\$ is to have (n-1) tails and then a head, this arrives with a probability $\left(\frac{1}{2}\right)^{n-1} \times \frac{1}{2} = \left(\frac{1}{2}\right)^{n}$. So for all n, we have:

$$E = \sum_{n=1}^{+\infty} \left(\frac{1}{2}\right)^{n} \times 2^n = \sum_{n=1}^{\infty} 1 = +\infty$$

2.2 Personal investment

I would personnally pay 2$ to play this game, then I cannot lose anything. Moreover, I think that the first head will generally happen in the first 3 tosses, so it is not worth paying more.

2.3 Expected utility

The formulae is:

$$EU = \sum_{n=1}^{\infty} p_n U_n$$

So we have:

$$EU = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n} \times (alog_2(n) + b) = \sum_{n=1}^{\infty} \left(\frac{a}{2^n} log_2(n) + \frac{b}{2^n}\right)$$

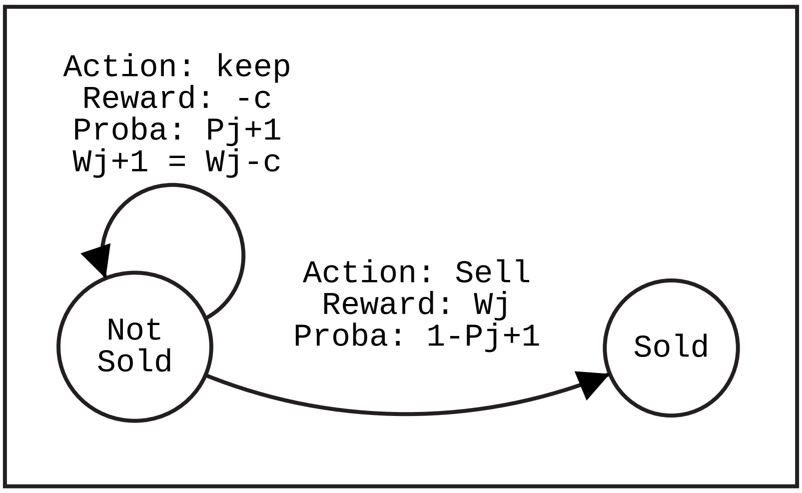
$$EU = a\sum_{n=1}^{\infty} \frac{log_2(n)}{2^n} + b\sum_{n=1}^{\infty} 2^{-n} = a\sum_{n=1}^{\infty} 2^{-n} log_2(n) + b\left(\frac{1}{2}.\frac{1-\left(\frac{1}{2}\right)^{+\infty}}{1-\frac{1}{2}}\right)$$

$$EU = a\sum_{n=1}^{\infty} 2^{-n} log_2(n) + b$$

3 Markov Decision Processes

The definition of the problem using the Markov Decision Processes is:

- States: House sold, House not sold
- Actions: Sell the house, Keep the house for the next day
- Reward: w_j if the house is sold, -c if the house is kept



4 Programming questions: Monte Carlo

4.1 Implementation

To implement the Monte Carlo, I have generated all my possible move. For each move, I have generated all the possible move of the opponent. For each of his move, I have evaluated the board as follows:

- I have summed all the distance between the middle of the board and my dies: D_{my}
- I have summed all the distance between the middle of the board and the opponent dies: D_{opp}
- I have substracted these values: $D_{my} - D_{opp}$
- I have average this quantity for all the possible move of the opponent: $\frac{D_{my}-D_{opp}}{N}$

I finally choose the move with the minimum average.

4.2 Results

My Monte Carlo player has played against the Random player and the Minimax player, he has always won. I can even say that my Monte Carlo player is better than me !